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Fictitious charges arrangement determination in Charge Simulation Method

INTRODUCTION

A determination of electrostatic field at the dielectric body mostly requires solving the Laplace's equation. But, in many cases, the physical systems are so complex that analytical solutions are difficult or even impossible to be found. Fortunately, with the application of the computer, it is now possible to use simple numerical approximation methods to solve more complex problems in a matter of a few minutes.

One of the mostly used methods for numerical electrostatic problems solving is Charge Simulation Method (CSM). Basic idea of this method is replacing the existing electrodes by fictitious charges (FC) chosen in certain order and placed inside the electrodes volumes. The unknown values of these charges can be determined by solving system of linear equations. The system of equations is formed in that way to satisfy boundary conditions on the electrodes surfaces.

After solving this system, the unknown values of the fictitious charges can be determined. Using standard electrostatic formulas the potential and the electric field strength can be calculated.

The correct choice of the type of fictitious charges is very important, especially with respect to the realized accuracy and convergence with the number of the fictitious charges. The most commonly used fictitious charges are point, line and ring charges.

Also, it is very important to determine the position of fictitious charges. The commonly posed questions are: How to place the fictitious charges, which are their optimal number, how to choose matching points? The answers on these questions depend on the experience of the investigator. But, some unwritten rule is that the number of fictitious charges can't be too large or the relative distance between the fictitious charges and the matching points can't be too small, because the system of equations could be ill conditioned.

CHARGE SIMULATION METHOD APPLICATION

In this paper, an optimal position of fictitious charges for two simple examples is determined. The analytical solutions exist for the problems of the cavity in dielectric medium and the two-layers dielectric cylinder in the homogeneous electric field. Fictitious charges are line charges placed at the cylindrical surfaces with circular cross-section. The radius of cylindrical surfaces is determined by coefficient f. At the boundary surfaces the boundary conditions for potential and the electric field strength should be satisfied. In that way a system of linear equations is formed. Solving this system, the potential and the electric field strength can be determined.
Changing the position of the fictitious charges and comparing the calculated electric field strength values with the correct values, the optimal position of the fictitious charges can be determined.

EXAMPLE I

A cylindrical cavity with circular cross-section of radius a is placed in a homogeneous dielectric medium with permittivity ε, Fig.1. It is necessary to determine the electric field strength inside the cavity. The whole system is placed in homogeneous electrostatic field, $\boldsymbol{E}_0 = E_0\hat{\boldsymbol{x}}$.

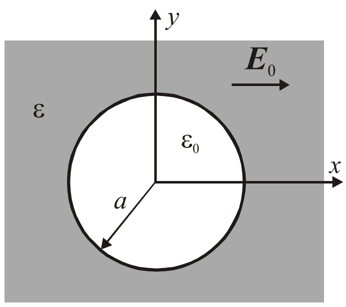


Fig.1 – *Cylindrical cavity in homogeneous dielectric medium*

An analytical solution for this problem exists. It is obtained after solving Laplace's equation,

$$\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right)+\frac{1}{r}\frac{\partial^2\varphi}{\partial\theta^2}=0\,, \tag{1}$$

in that way to satisfy boundary conditions:

$$\varphi(r=a-0)=\varphi(r=a+0) \text{ and} \tag{2}$$

$$\varepsilon_0\left.\frac{\partial\varphi}{\partial r}\right|_{r=a-0}=\varepsilon\left.\frac{\partial\varphi}{\partial r}\right|_{r=a+0}. \tag{3}$$

At the large distance from the cavity the potential is practically equal to the potential of homogeneous field

$$\lim_{r>>a} \varphi = \varphi_0 = -E_0 x = -E_0 \cos\theta \,. \tag{4}$$

Solving formed differential equation the potential is:

$$\varphi = \begin{cases} -\dfrac{2\varepsilon}{\varepsilon+\varepsilon_0} E_0 r\cos\theta, \; r \le a \\ -E_0\left(r + \dfrac{a^2}{r}\dfrac{\varepsilon-\varepsilon_0}{\varepsilon+\varepsilon_0}\right)\cos\theta, \; r \ge a. \end{cases} \tag{5}$$

Inside the cavity the electric field is homogeneous

$$E = \frac{2\varepsilon_r}{\varepsilon_r + 1} E_0 \ge E_0 \,. \tag{6}$$

For $\varepsilon_r = 3$ the electric field strength is $E/E_0 = 1.5$.

Using the Charge Simulation Method the whole system is divided in two systems, Fig.2 [5].

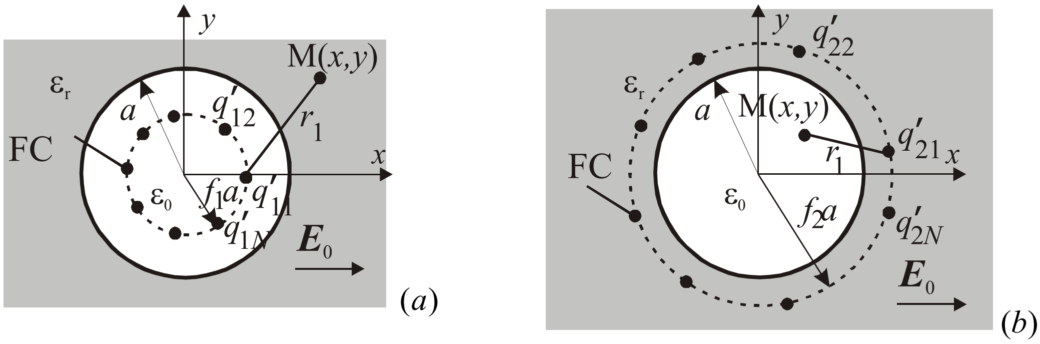


Fig.2 – *Charge Simulation Method application*

The potential is continuous and the normal components of dielectric displacement are equal at the boundary surface.

In Table I, the electric field strength values for different number of fictitious charges are shown. The values of coefficient f are known, $f_1 = 0.7$ and $f_2 = 1.2$.

Table I. *Electric field strength, E/E_0, inside the cavity for $\varepsilon_r = 3$ and different number of fictitious charges.*

N	E/E_0
5	0.90212907938
10	1.27983669740
30	1.49572751017
50	1.49989337464
80	1.49999956051
100	1.49999998862

There is a very good convergence of the results depending on the number of fictitious charges. But, the chosen values of coefficient f don't give an exact solution that has been obtained by analytical method, $E/E_0 = 1.5$. Because of that, a procedure for determination an optimal positions of fictitious charges is applied.

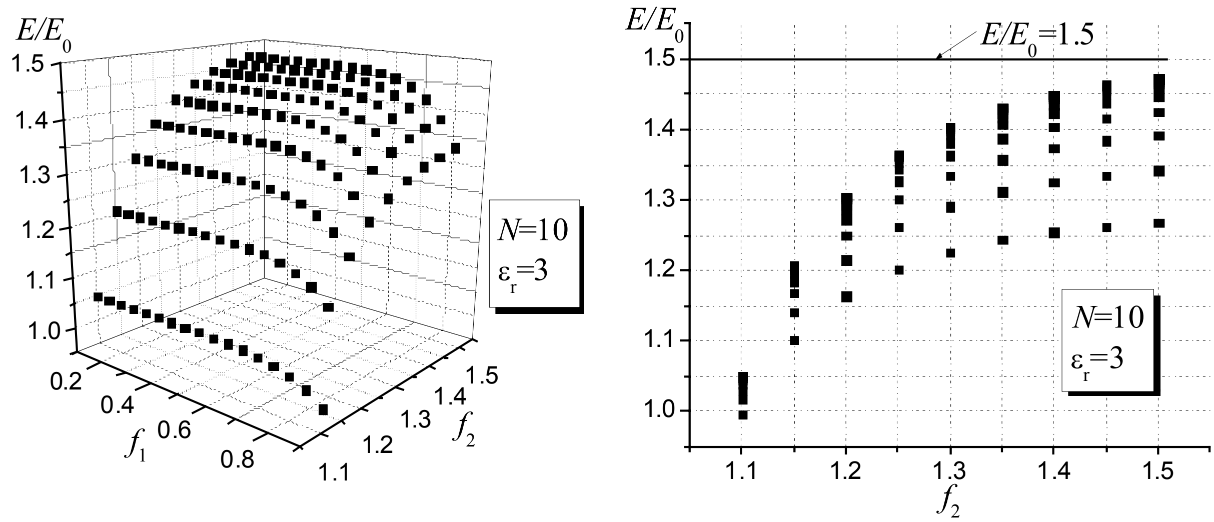
Using this procedure, the number of fictitious charges is known, but coefficients f change their values from $0.1 < f_1 < 0.9$ and $1.1 < f_2 < 1.5$, with a step 0.1. For every value of coefficient f the electric field strength value has been calculated and has been compared with the exact value. In Table II the optimal values of the coefficient f are shown, where the number of fictitious charges is $N = 70$.

Table II. *Optimal position of fictitious charges, for* $\varepsilon_r = 3$ *and* $N = 70$.

f_1	f_2	E/E_0
0.10	1.5	1.50000000000
0.20	1.5	1.50000000000
0.30	1.5	1.50000000000
0.40	1.5	1.50000000000
0.50	1.5	1.50000000000
0.60	1.5	1.50000000000

The discrete electric field strength values inside the cavity for different values of coefficients f_1 and f_2 and different number of fictitious charges N are shown in Fig.3. In Fig.3(a) a 3D presentations of electric field values are shown. Because of better view, obtained solutions are presented in $E - f_2$ plane in Fig.3(b).

It can be seen that with increasing number of fictitious charges, the electric field strength values are grouping round the electric field exact value.



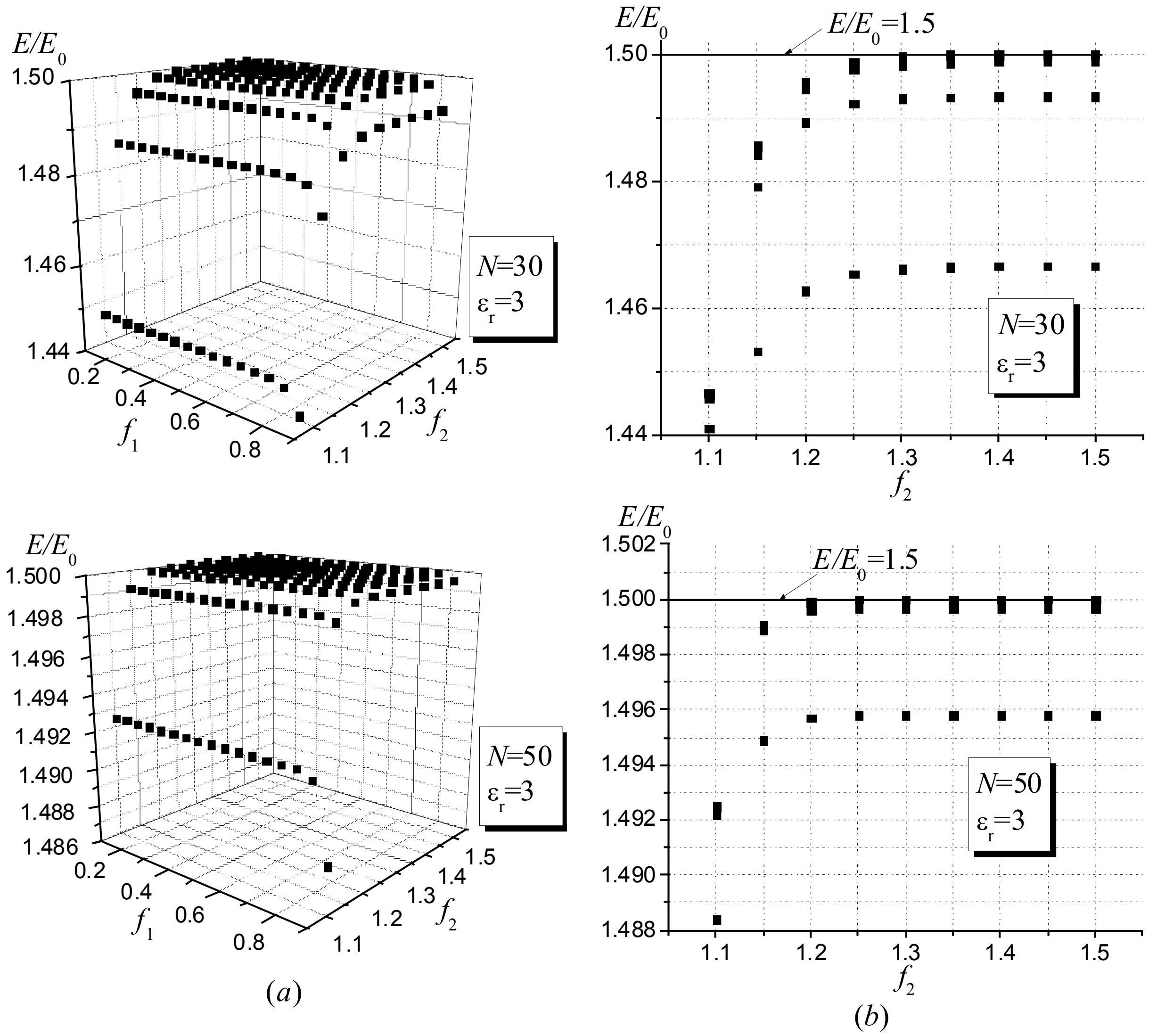


Fig.3 – *Discrete values of electric field inside the cavity for different values of coefficient* f *and different number of fictitious charges*

In Table III the results obtained using CSM have been compared with program packages femm and FEMLAB results as well as with analytical solution. There is a very good agreement of those results (an error rate less than 1.6%). Program packages femm and FEMLAB use the Finite Element Method. The program packages input values are identical to the CSM input values.

Table III
Results comparing.

Applied method	E/E_0
Analytical solution	1.50000
Charge Simulation Method ($N = 100$, $f_1 = 0.7$, $f_2 = 1.3$)	1.50000
Finite Element Method (femm 4.0)	1.47522
Finite Element Method (FEMLAB)	1.47679

EXAMPLE II

Two-layers dielectric cylinder is placed in a homogeneous electrostatic field, $\boldsymbol{E}_0 = E_0\hat{\boldsymbol{x}}$, Fig.4.

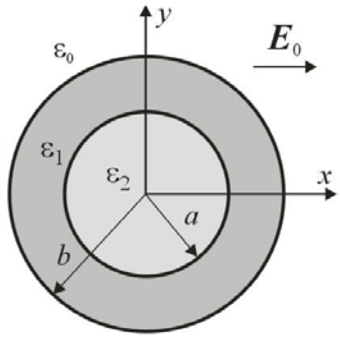


Fig.4 – *Two-layers dielectric cylinder*

The analytical solution for the electric field inside the cylinder (for $|x| < a$) is

$$E = gE_0 \tag{7}$$

where
$$g = \frac{4\varepsilon_0\varepsilon_1}{(\varepsilon_0 + \varepsilon_1)(\varepsilon_1 + \varepsilon_2) + \left(\dfrac{a}{b}\right)^2 (\varepsilon_0 - \varepsilon_1)(\varepsilon_1 - \varepsilon_2)} \tag{8}$$

is a “protection factor” that defines the penetrated electric field inside the body through the shield that has the permittivity ε_1.

For $a/b = 0.5$, $\varepsilon_{r1} = 5$ and $\varepsilon_{r2} = 2$, the electric field is homogeneous and its value is $E/E_0 = 0.512820$.

Using Charge Simulation Method the whole system is divided in three systems, Fig.5.

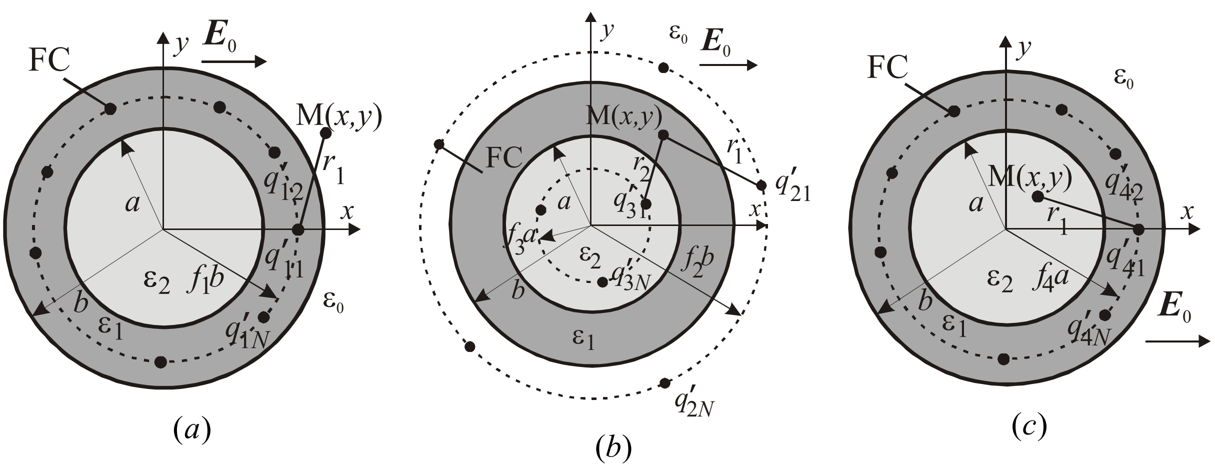


Fig.5 – *Charge Simulation Method application*

The fictitious charges are line charges, placed at the cylindrical surfaces with circular cross-section with radius $f_1 b$, $f_2 b$, $f_3 a$ and $f_4 a$ $(0 < f_1 < 1,\ f_2 > 1,\ 0 < f_3 < 1$ and $f_4 > 1)$.

In Table IV the electric field value in two points along $x-$axis, for different number of fictitious charges is shown. The input values are: $a/b = 0.5$, $\varepsilon_{r1} = 5$ and $\varepsilon_{r2} = 2$. The values of coefficient f are: $f_1 = 0.899$, $f_2 = 1.2$, $f_3 = 0.9$ and $f_4 = 1.2$.

Table IV

Electric field, E/E_0, in point $\mathrm{M}(x/b, y/b)$ for different number of fictitious charges.

N	$\mathrm{M}(0,0)$	$\mathrm{M}(0.75,0)$
10	0.359833	0.288902
30	0.507701	0.299417
50	0.512737	0.291860
80	0.512821	0.290653
100	0.512820	0.290605
120	0.512820	0.290599

The calculated values of electric field, when the number of fictitious charges is known and values of coefficient f are different, have been compared with analytical solution. In that way, optimal positions of fictitious charges have been determined. Those results are shown in Table V. The electric field values for different number of fictitious charges and different values of coefficient f are shown in Fig. 6.

Table V

Optimal values of coefficient f, for $a/b = 0.5$, $\varepsilon_{r1} = 5$, $\varepsilon_{r2} = 2$ and $N = 80$.

f_1	f_2	f_3	f_4	E/E_0
0.7	1.5	0.1	1.4	0.51282051282
0.7	1.5	0.2	1.4	0.51282051282
0.7	1.5	0.3	1.4	0.51282051282
0.7	1.5	0.4	1.4	0.51282051282
0.7	1.5	0.5	1.4	0.51282051282
0.7	1.5	0.6	1.4	0.51282051282

In Table VI, the CSM calculated results for electric field, when the number of fictitious sources of the each system is $N = 80$, have been compared with analytical solution and the program packages results. Input values are identical to Charge Simulation Method input values, and electric field in program packages is $E_0 = 100\ \mathrm{V/m}$. Fig.7 shows the electric field distribution obtained using the program package femm 4.0.

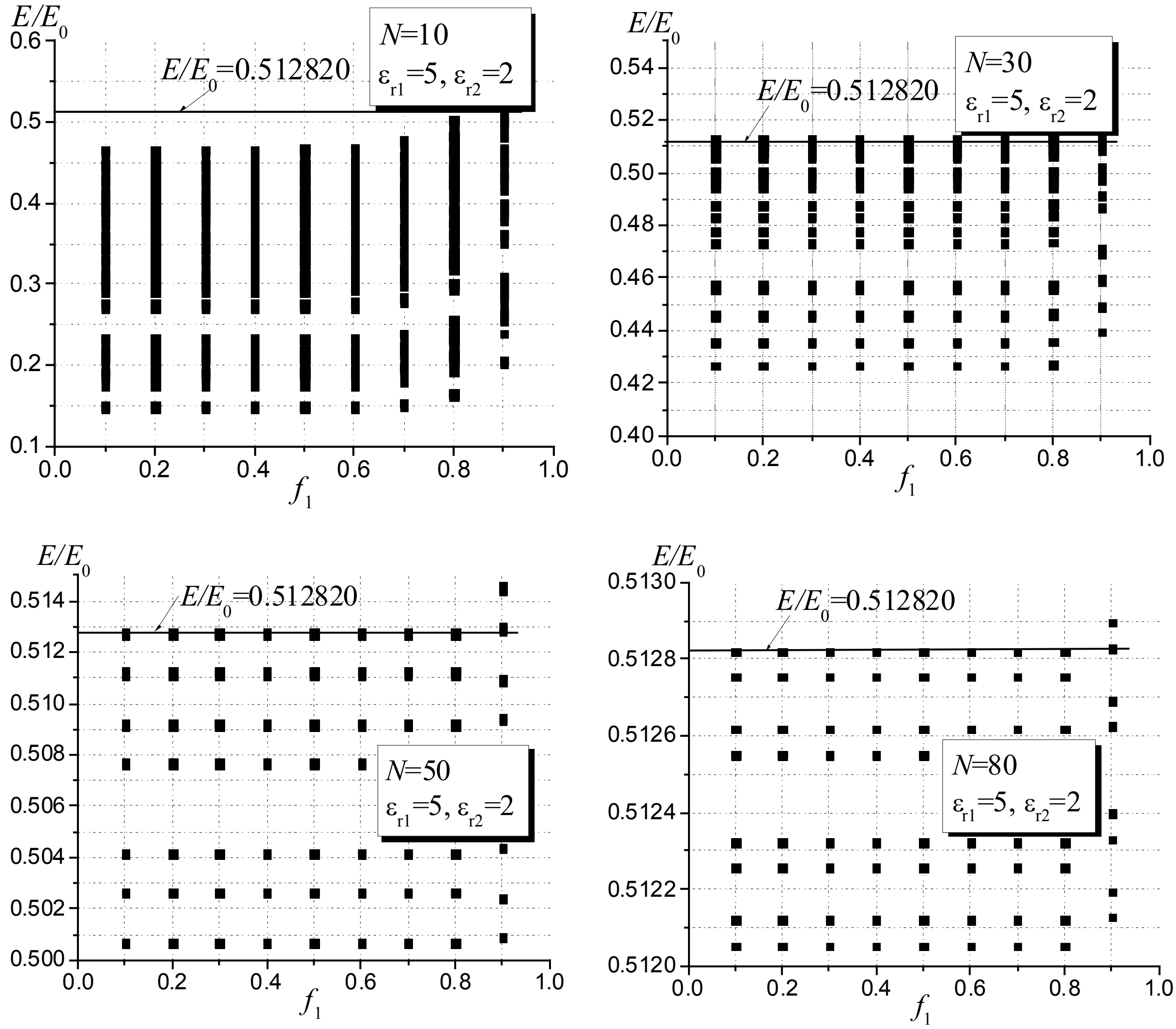


Fig.6 – *Discrete values of electric field inside the cylinder for different values of coefficient f and different number of fictitious charges*

Table VI

Electric field results comparing, E / E_0.

Applied method	M(0,0)	M(0.75,0)
Analytical method	0.512820	0.290598
Charge Simulation Method	0.512820	0.290605
Finite Element Method (femm 4.0)	0.523663	0.297639
Finite Element Method (FEMLAB)	0.522716	0.298253

CONCLUSION

In a last few years there has been developed a large number of optimisation methods, such as genetic algorithm. In this paper, an analytical solution for considered problems is known. Changing the position of the fictitious charges and comparing the calculated electric field strength values with the correct values, the optimal position of the fictitious charges can be determined. The applied method in this paper shows that the relative distance between the fictitious charges and the matching points can’t be too small.

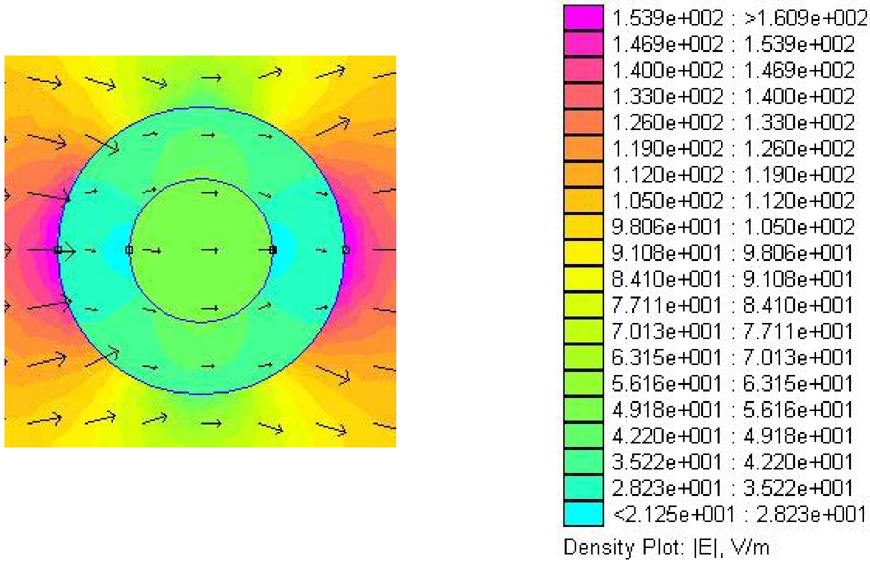


Fig.7 – *Electric field distribution (femm 4.0)*

Also, the calculated values have been compared with the program packages results. The obtained error is less than 2%. The applied program packages use Finite Element Method.

The number of fictitious charges can’t be too large because the system of equations could be ill conditioned. In those examples a good convergence of the results is obtained for 50 fictitious charges.

References:

[1] M. T. Perić, S. R. Aleksić: "Two layers dielectric cylinder in homogeneous field" (in Serbian), XIII Telecommunications forum TELFOR 2005, CD Proceedings, section 7.8: AEE, Belgrade, Serbia and Montenegro, November 2005.

[2] H. Brauer: “Computation of electromagnetic fields using integral methods”, International PhD - Seminar “Computation of Electromagnetic Field”, Budva, Serbia and Montenegro, September 2004, pp. 201-212.

[3] R. Nishimura, K. Nishimori, N. Ishihara: "Determining the arrangement of fictitious charges in charge simulation method using genetic algorithms", Journal of Electrostatics, Vol. 49, Issues 1-2, May 2000, pp. 95-105.

[4] J. V. Surutka, D. M. Veličković: “Some improvements of the Charge Simulation Method for Computing Electrostatic Fields”, Bulletin LXXIV de l'Academie Serbe des Sciences et des Arts, Class des Sciences technique, No. 15; 1981, pp. 27-44.

[5] M. T. Perić: "Penetration of electric field into hollow bodies" (in Serbian), Master thesis, Faculty of Electronic Engineering, Niš, February 2006

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